Minds vs. AIs

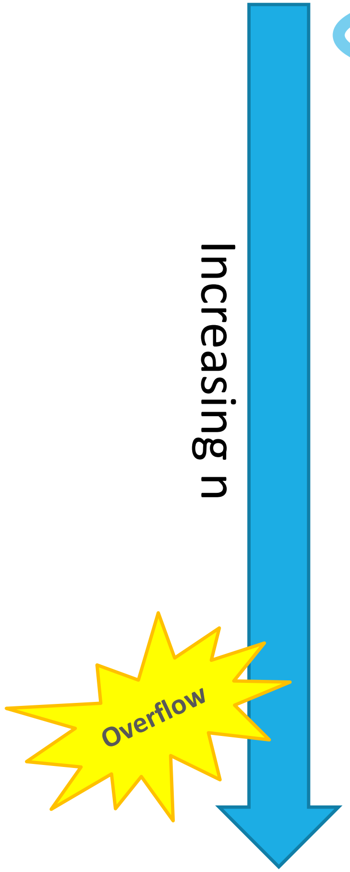
GIUSEPPE BONACCORSO

Warming up…

What does this mean? ***(Not for a mathematician)***

$$\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e$$

For any computer (actual or potential)

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Approximation(e) = 2.71828 18284 59045 23536 02874 71352 66249 77572 47093 69995 95749 66967 62772 40766 30353 54759
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93163 68892 30098 79312 77361 78215 42499 92295 76351 48220 82698 95193 66803 31825 28869 39849 64651 05820 93923 98294
88793 32036 25094 43117 30123 81970 68416 14039 70198 37679 32068 32823 76464 80429 53118 02328 78250 98194 55815 30175
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47910 14590 40905 86298 49679 12874 06870 50489 58586 71747 98546 67757 57320 56812 88459 20541 33405 39220 00113 78630
09455 60688 16674 00169 84205 58040 33637 95376 45203 04024 32256 61352 78369 51177 88386 38744 39662 53224 98506 54995
88623 42818 99707 73327 61717 83928 03494 65014 34558 89707 19425 86398 77275 47109 62953 74152 11151 36835 06275 26023
26484 72870 39207 64310 05958 41166 12054 52970 30236 47254 92966 69381 15137 32275 36450 98889 03136 02057 24817 65851
18063 03644 28123 14965 50704 75102 54465 01172 72115 55194 86685 08003 68532 28183 15219 60037 35625 27944 95158 28418
82947 87610 85263 98139 55990 06737 64829 22443 75287 18462 45780 36192 98197 13991 47564 48826 26039 03381 44182 32625
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03117 34392 07321 13908 03293 63447 97273 55955 27734 90717 83793 42163 70120 50054 51326 38354 40001 86323 99149 07054
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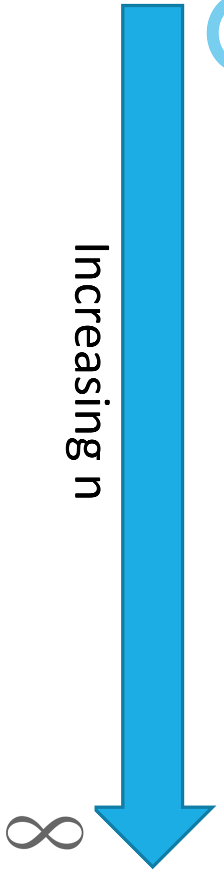
For a human being



e 2.71828 18284 59045 23536 02874 71352 66249 77572 47093 69995 95749 66967 62772 40766 30353 54759 45713 82178 52516
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58048 96690 62951 19432 47309 95876 55236 81285 90413 83241 16072 26029 . . .

This is the real difference!

The starting (and ending) point

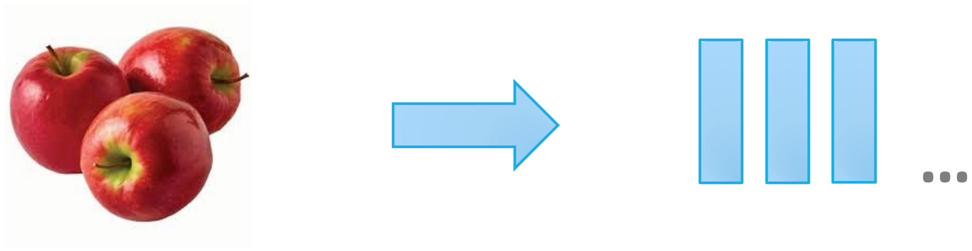
1 + 2 + 3 + … can really continue forever?

Why can we *(easily)* manage infinity and undecidability?

Is the proposition A true or false? I can't decide!

Counting… from 1 to…

It's "easy" to observe that with 3 apples we can create a 1:1 correspondence with an abstract set:

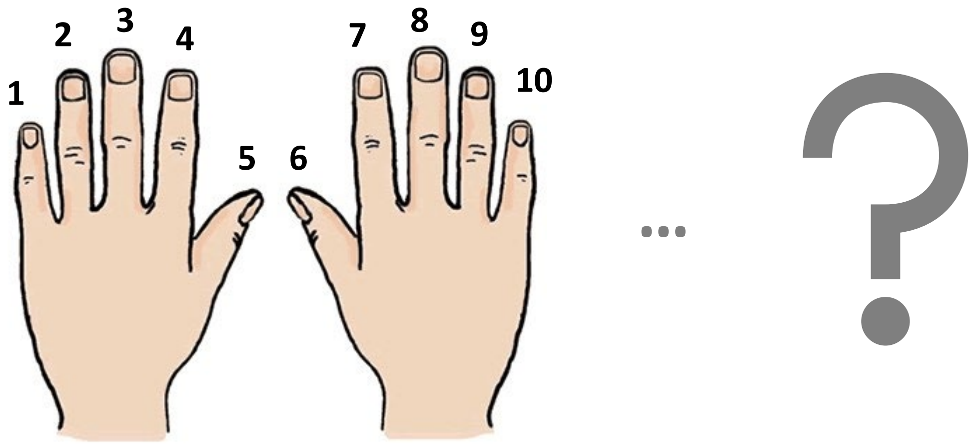


This set is ordered, and we can give a name to its elements: 1, 2, 3, … (i.e., $\mathbb{N} = \{1, 2, 3, \dots\}$)

How many elements does $\mathbb{N}$ contain?

From your hands to the stars…

Given the first element 1 and the generic element $n \in \mathbb{N}$, any other element is obtained using the addition $n + 1 \in \mathbb{N}$



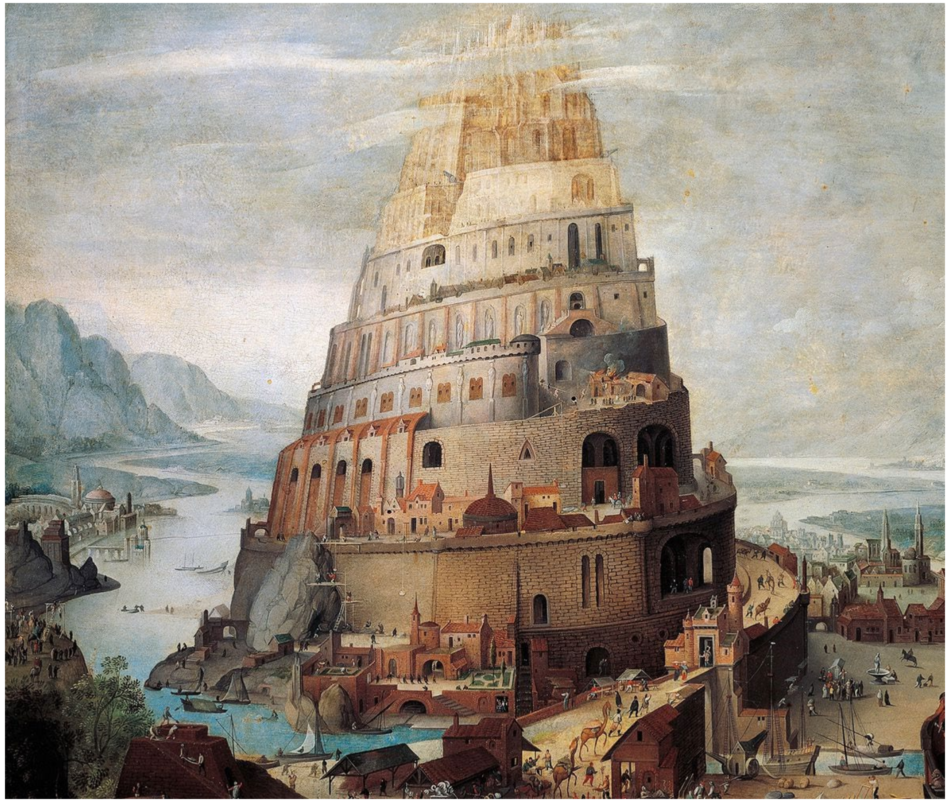
It's necessary a mental "*shift*"!

An additional abstraction to step from a finite representation to something different…

A shift called language…

The most powerful invention made by human beings is language!

"The limits of my language means the limits of my world." (L. Wittgenstein)



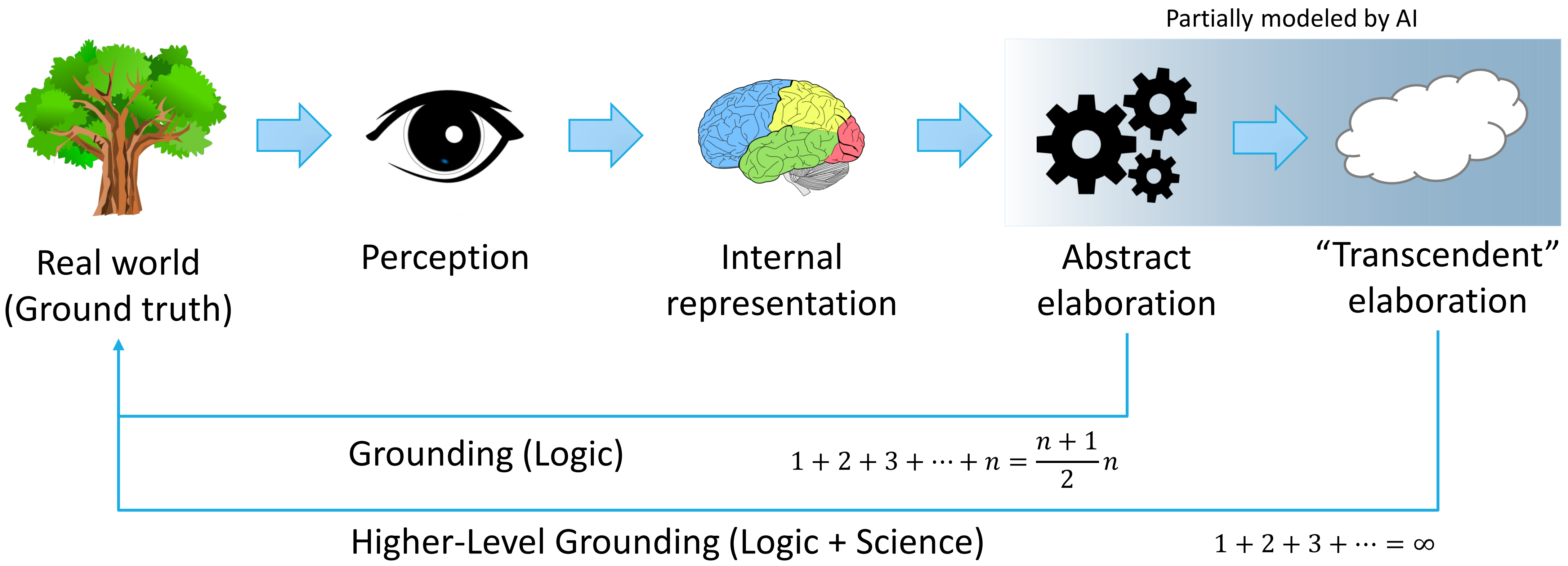
Reaching the stars

1 belongs to $\mathbb{N}$, $2 = 1 + 1, 3 = 2 + 1, \ldots$ I can continue this process till… I die!

…But, if after my death, my child will continue from my last number n_1, will she find an end? She can ask her child, and so on…

Even if I can never reach it, there's a "limit", not a number… Something different, greater than any $n \in \mathbb{N}$!

Language + Logic + Science = Mind



Language (+ Logic) = Coherent reality

Language only:

=

- The tree in on the left. The lion is on the right. ✓
- The lion can walk away from the tree. ✓
- The tree will eat the lion. ✓

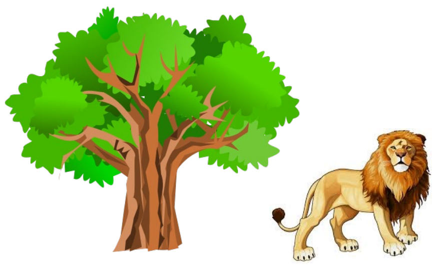
Language + Logic:

=

- The tree cannot move. The lion can. ✓
- No trees can eat any lion. ✓
- All lions can run away. ✗

Language + Logic + Science = Reality

Language + Logic + Science help modeling the reality, creating **internal representations**, and correct **abstractions**:



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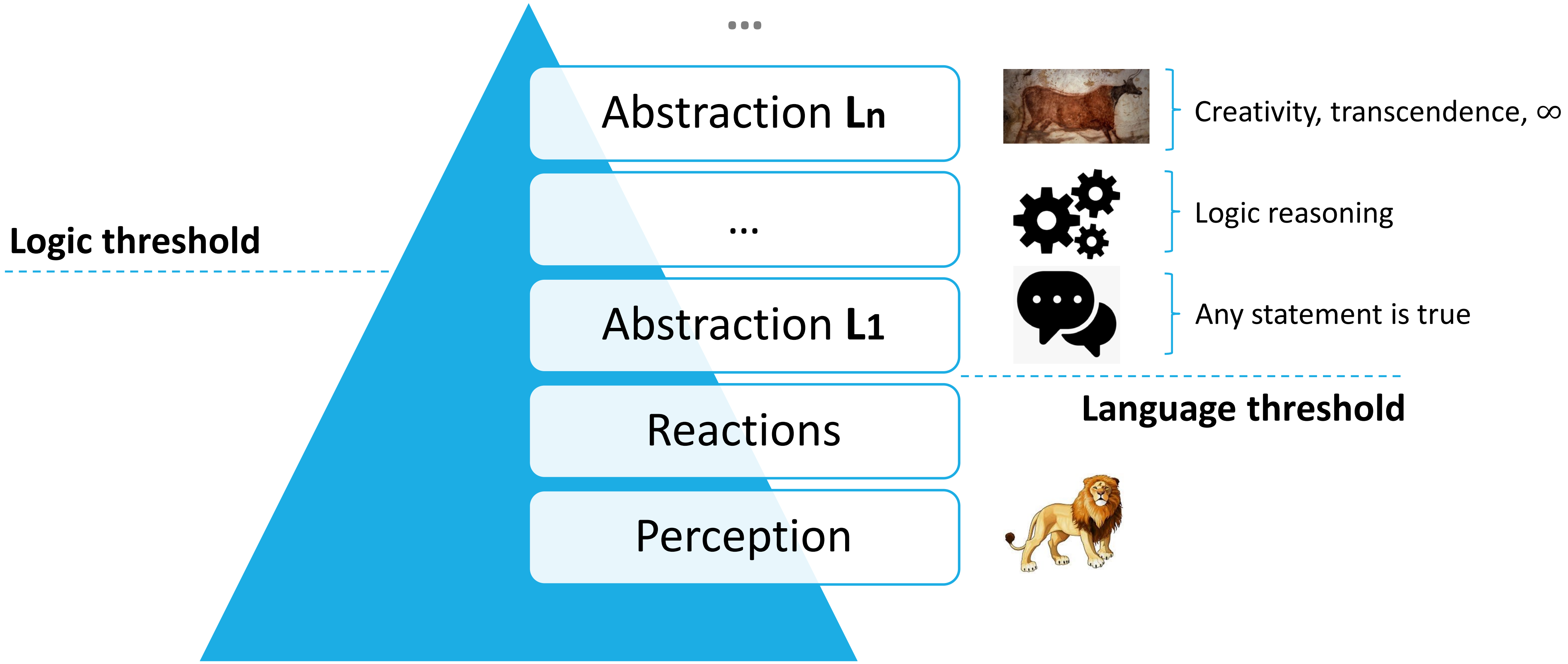
An adult lion is on average too heavy to stay on a branch of the tree. ✓

Another body with the same mass of an adult lion will break a branch of the tree. ✓

It's reasonable to imagine the existence of a tree whose branches can hold a lion. ✓

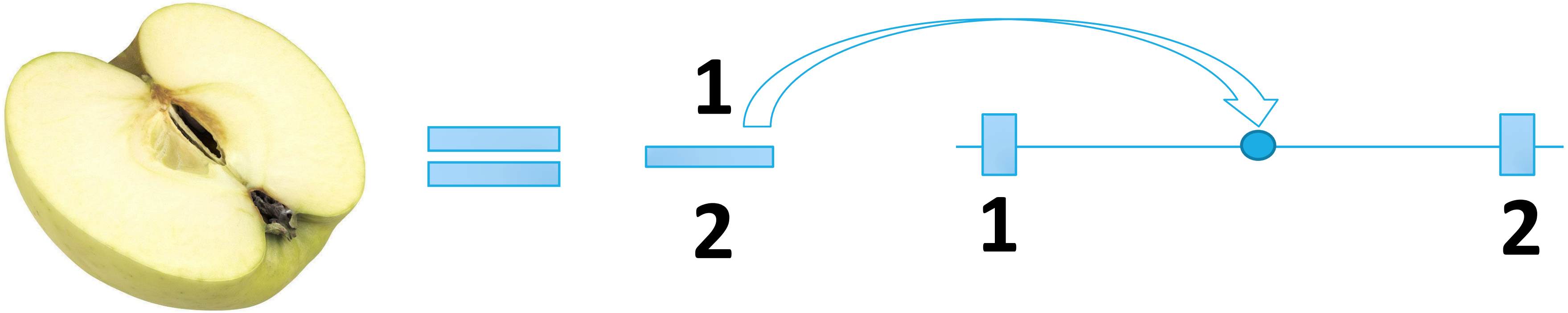
↓ Abstraction level

An “infinite” tower of abstractions



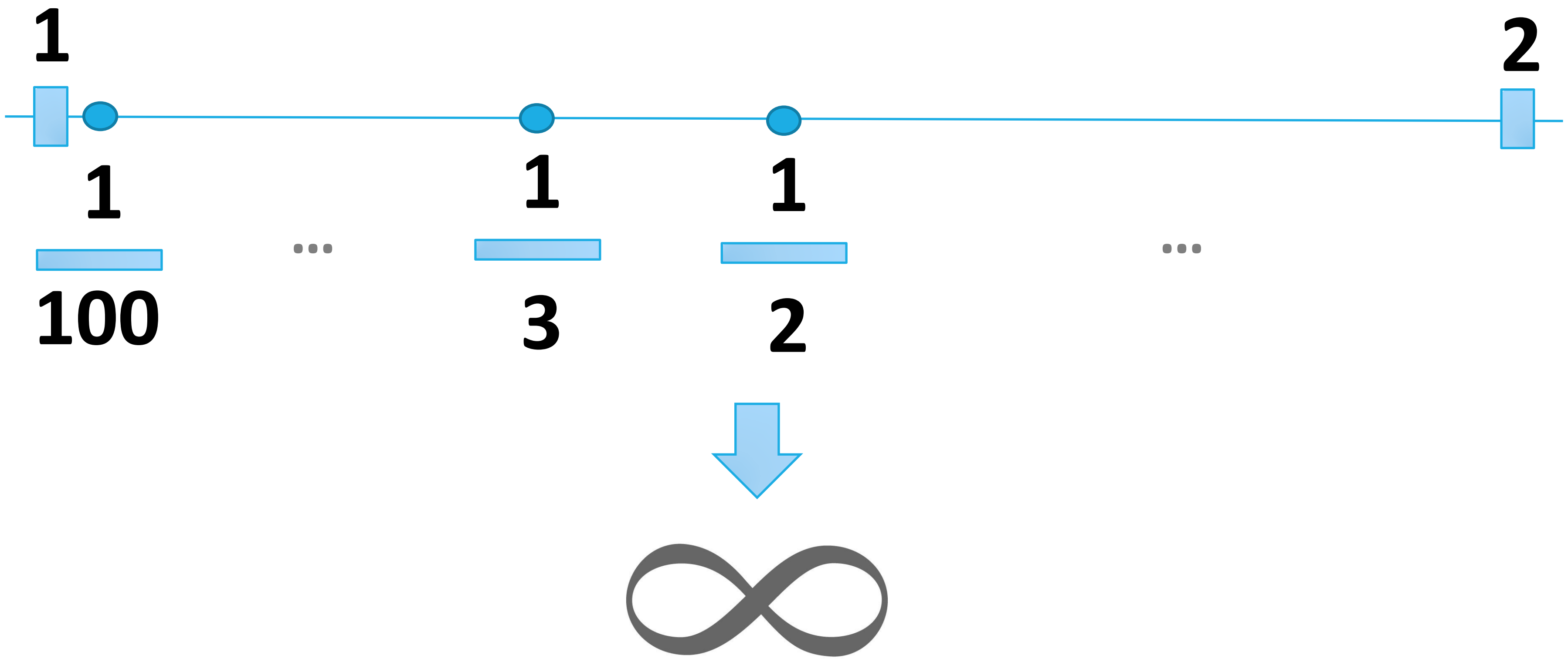
From the stars to the galaxies…

How can we represent half an apple? Using rational numbers $x \in \mathbb{Q}$:



How many fractions are there between 1 and 2?

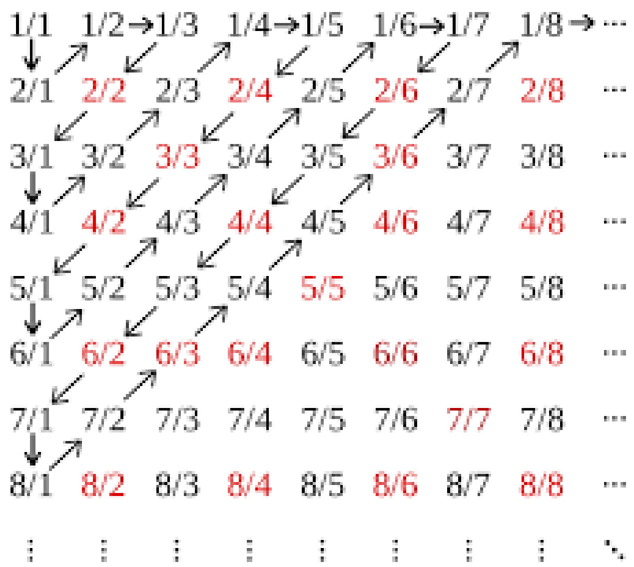
A crowd of rational numbers!



Galaxies are smaller than expected

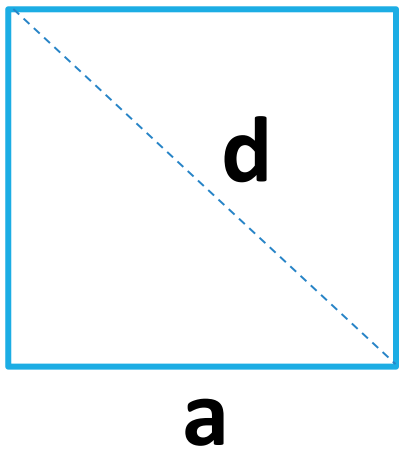
Let’s rephrase the question. Has the infinity of $\mathbb{N}$ the same “size” of the infinity of $\mathbb{Q}$?

Even if it’s counterintuitive, a clever trick allowed to prove that $\mathbb{N}$ and $\mathbb{Q}$ have the “same size”!



Pythagoras is nasty!

Given a square with side equal to $a = 1$, the diagonal is $d = \sqrt{2}$. Does $\sqrt{2} \in \mathbb{Q}$?



If $\mathrm{d} = \frac{x}{y}$ $with\ x, y \in \mathbb{N}$ and without common factors:

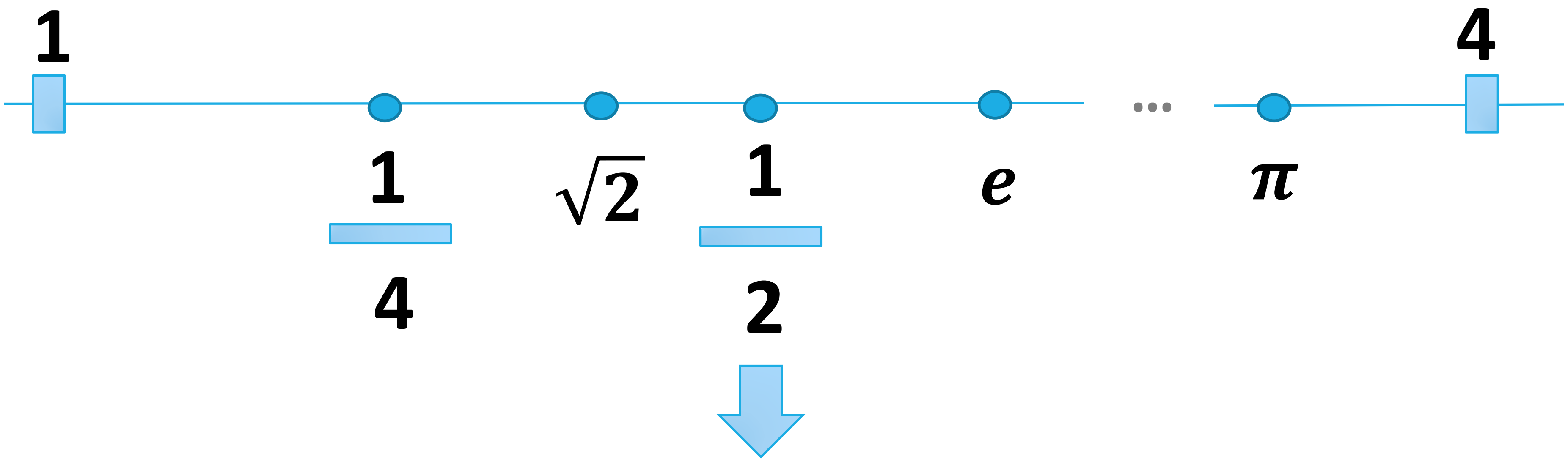
$$\frac{x^2}{y^2} = 2 \quad \Rightarrow \quad x^2 = 2y^2$$

It's easy to check that if x is odd, $x^2$is also odd, but $2y^2$ is even. In a similar way, it's possible to see that x cannot be even.

$\sqrt{2}$ is not rational! Hence $\sqrt{2} \notin \mathbb{Q}$. Where is it?

From galaxies to the Universe...

$\sqrt{\mathbf{2}}$ was defined as **irrational**, giving birth to a new set $\mathbb{R} = \mathbb{Q} \cup \{\sqrt{2}, \pi, e, \dots\}$ called **real** numbers:



Of course we need a ask the same question.
Does $\mathbb{R}$ have the same "size" as $\mathbb{Q}$?

From galaxies to the Universe…

$\sqrt{\mathbf{2}}$ was defined as irrational, giving birth to a new set $\mathbb{R} = \mathbb{Q} \cup \{\sqrt{2}, \dots\}$

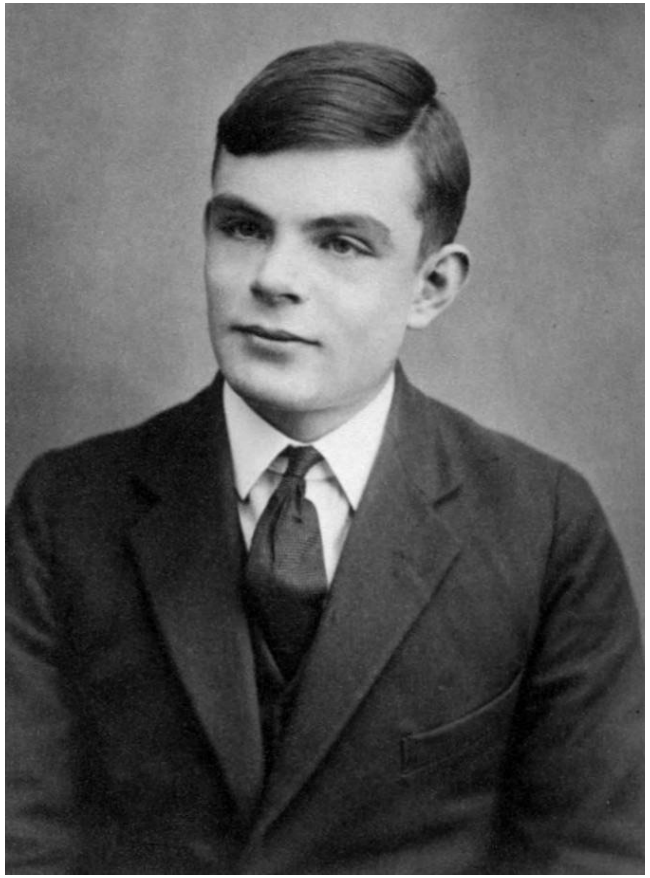
George Cantor proved that the "size" of $\mathbb{R}$ is "**much larger**"* than size of $\mathbb{N}$ and $\mathbb{Q}$, which are defined as **countable**.

$\mathbb{R}$, instead, has the power of continuity, i.e., it is **continuous**.

Cantor also hypothesized that there are no other infinities between the previous two.

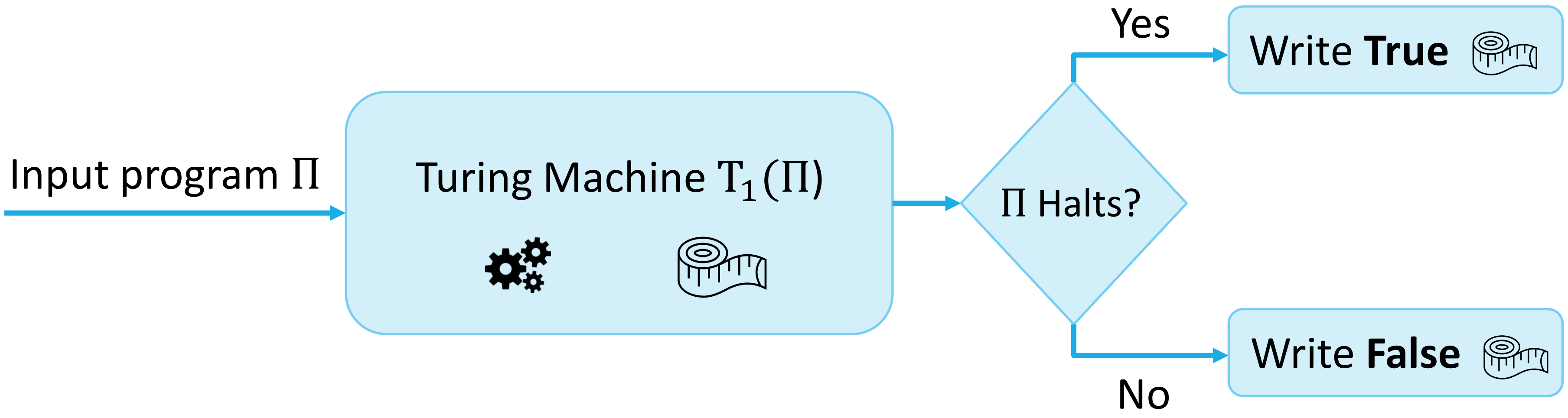
* We are omitting all mathematical formalism

And now… Let’s wake up Turing!



I always know when a program stops!

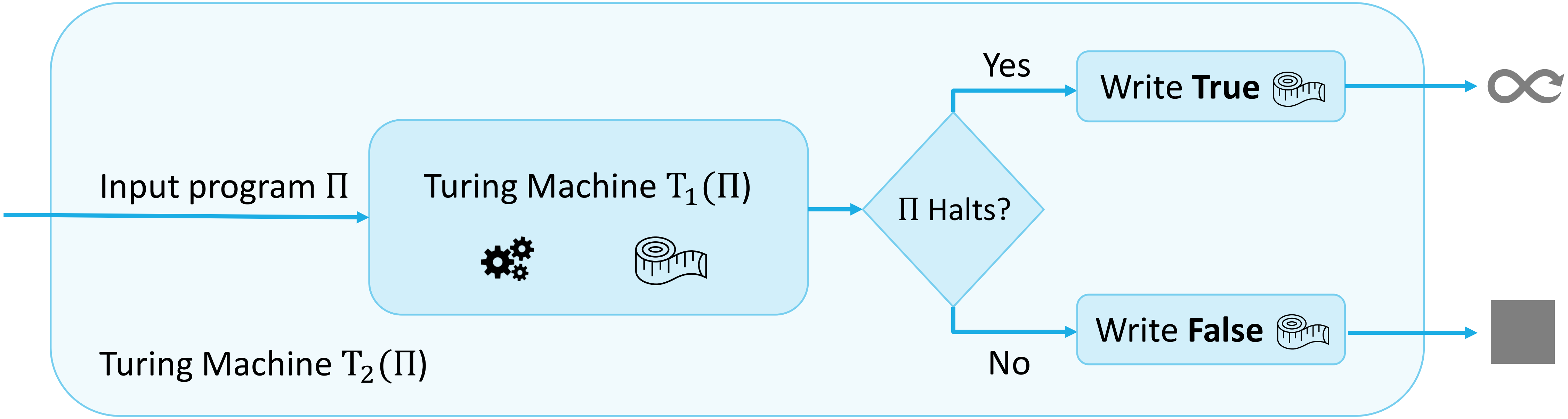
Suppose to build a Turing Machine* $T_1(\Pi)$ that can always decide whether the input program Π halts write either **True** or **False** on the tape (supposed to be infinite):



* The notation is voluntarily over-simplified

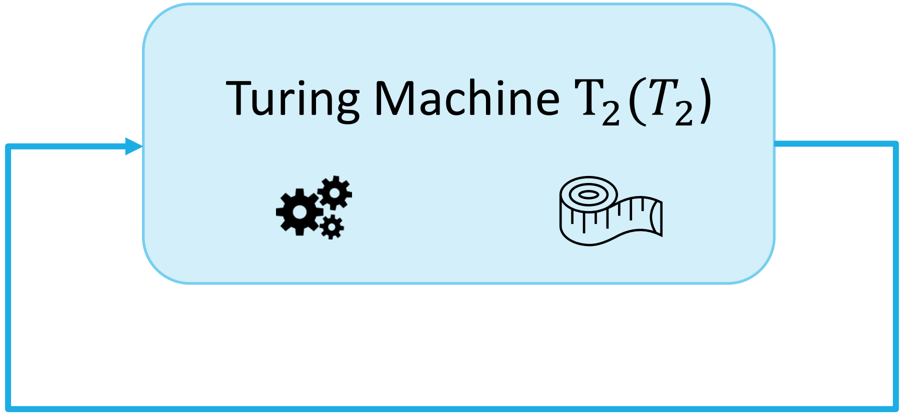
We can create and *ad-hoc* $\mathrm{T}_2(\Pi)$

When the sub-machine $\mathrm{T}_1(\Pi)$ writes **True**, $\mathrm{T}_2(\Pi)$ will loop forever and vice versa:



What’s the output of $\mathrm{T}_2(T_2)$?

Everything breaks!



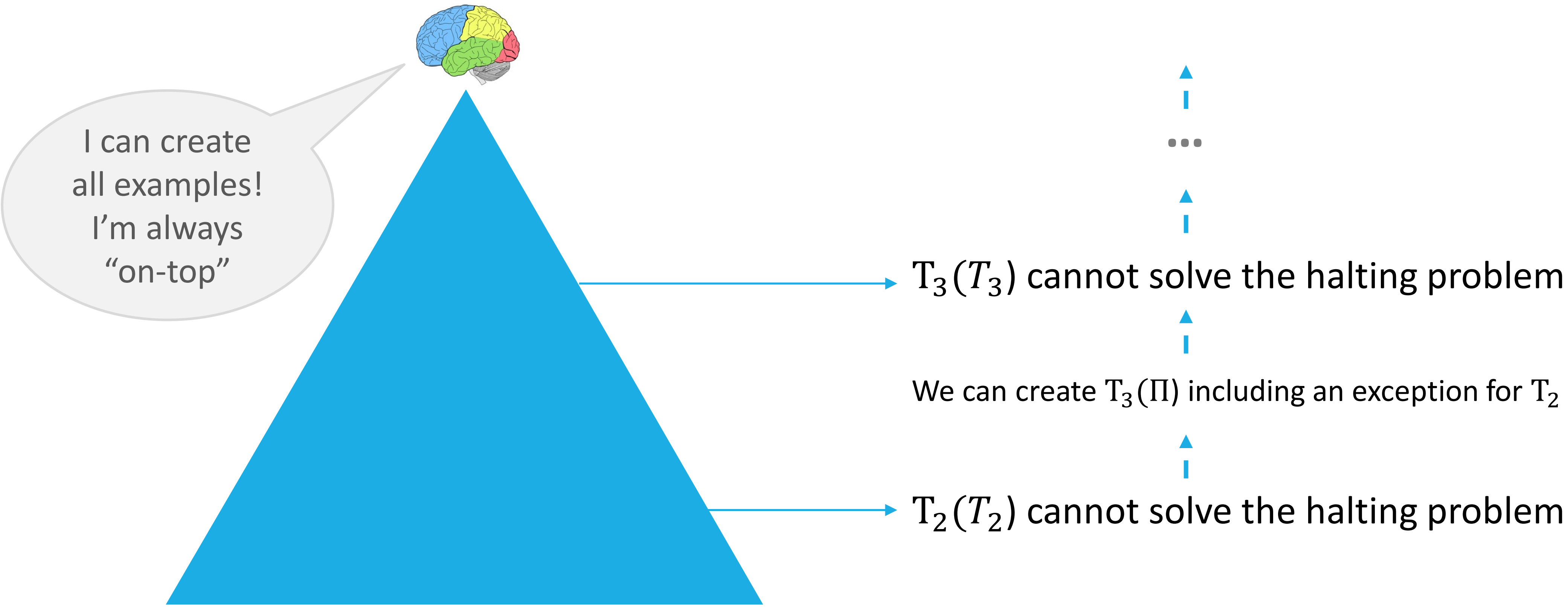
If T_2 writes **True**, $\mathrm{T}_2(T_2)$ loops forever

If T_2 writes **False**, $\mathrm{T}_2(T_2)$ halts

The problem in undecidable!

Let’s return into our tower

We **can** always create new machines, but $\mathrm{T}_n(T_n)$ won’t ever solve the halting problem!



Church-Turing Theses

Church-Turing Thesis

Any deterministic computing device is representable either through a Turing Machine or λ-Calculus.

Physical Church-Turing Thesis

Any deterministic procedure computable in finite time by any physical device is also computable by a Turing Machine.

Strong Church-Turing Thesis

Any deterministic procedure computable in polynomial time by any physical device is also computable in polynomial time by a Turing Machine.

The starting (and ending) point

Is human brain (mind) a Turing Machine?

If not, what kind of computing device is it?

Thank you!

I’m glad to entertain your questions!

Feel free to connect on LinkedIn for daily updates!